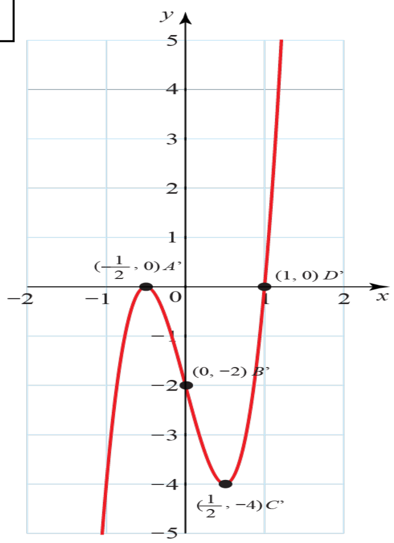
Advanced Subsidiary

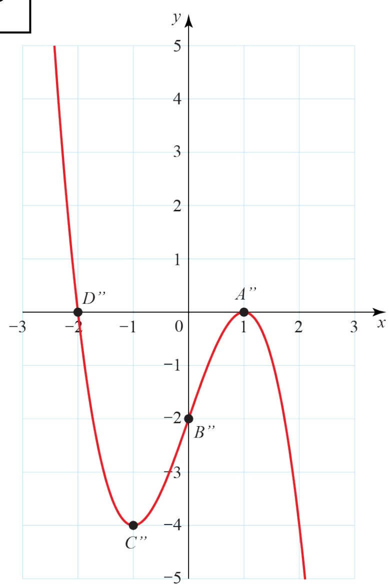
Paper 1: Pure Mathematics

PAPER D Mark Scheme

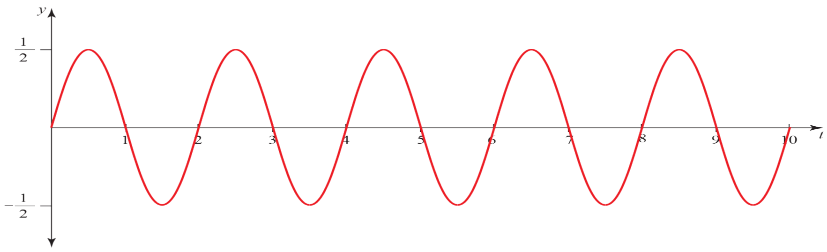
1a

Figure 3 	Evidence of attempt to show stretch of sf $\frac{1}{2}$ in x direction (e.g. one correct set of coordinates – not (0, –2)).	**M1**
	Fully complete graph with all points labelled. $(-1/2, 0)$, $(1, 0)$, $(0, -2)$, $(1/2, -4)$	**A1**
		2 marks

1b

Figure 4 	Evidence of attempt to show reflection in y axis (e.g. one correct set of coordinates – not (0, –2)).	**M1**
	Fully complete graph with all points labelled. $(-2, 0)$, $(1, 0)$, $(0, -2)$, $(1, -4)$	**A1**
		(2 marks)
		Total 4 marks

2 Makes an attempt to expand $\left(5-3\sqrt{x}\right)\left(5-3\sqrt{x}\right)$. Must be 4 terms (or 3 if $\sqrt{x}$ terms collected).	**M1**
Fully correct expansion $25-30\sqrt{x}+9x$ or $25-30x^{\frac{1}{2}}+9x$	**A1**
Writes $\sqrt{x}$ as $x^{\frac{1}{2}}$ (or subsequently correctly integrates this term)	**B1**
Makes an attempt to find $\int(25-30x^{\frac{1}{2}}+9x)\mathrm{d}x$. Raising x power by 1 at least once would constitute an attempt.	**M1**
Fully correct integration. $25x-20x^{\frac{3}{2}}+\frac{9}{2}x^2+C$ o.e.	**A1**
NOTE: Award all 5 marks for a fully correct final answer, even if some working is missing.	**Total 5 marks**
3 Correctly factorises. $\left(8^{x-1}-2\right)\left(8^{x-1}-16\right)=0$ (or for example, $\left(y-2\right)\left(y-16\right)=0$)	**M1**
States that $8^{x-1}=2$, $8^{x-1}=16$ (or $y = 2$, $y = 16$).	**A1**
Makes an attempt to solve either equation (e.g. uses laws of indices. For example, $\sqrt[3]{8}=2$ or $8^{\frac{1}{3}}=2$ or $\left(\sqrt[3]{8}\right)^4=16$ or $8^{\frac{4}{3}}=16$ (or correctly takes logs of both sides).	**M1**
Solves to find $x=\frac{4}{3}$ o.e. or awrt 1.33	**A1**
Solves to find $x=\frac{7}{3}$ o.e. or awrt 2.33	**A1**
NOTE: 2nd M mark can be implied by either $x-1=\frac{1}{3}$ or $x-1=\frac{4}{3}$	**Total 5 marks**

4a		
	Correct shape of sine curve through (0, 0).	**B1**
	Sine curve has max value of $\frac{1}{2}$ and min value of $-\frac{1}{2}$	**B1**
	Sine curve has a period of 2 (can be implied by 5 complete cycles) and passes through (1,0), (2,0),..., (10,0).	**B1**
		(3 marks)
4b	Student states that the buoy will be 0.4 m above the still water level 10 times.	**B1**
		(1 mark)
4c	Sensible and correct reason. For example: A buoy would not move up and down at exactly the same rate during each oscillation. The period of oscillation is likely to change each oscillation. The maximum (or minimum) height is likely to change with time. Waves in the sea are not uniform.	**B1**
	Award the mark for a different explanation that is mathematically correct. For example, stating that the buoy would not move exactly vertically each time.	**(1 mark)**
		(otal 5 marks
5	Attempts to differentiate.	**M1**
	$f'(x) = 3x^2 - 8x - 35$	**A1**
	States or implies that f(x) is increasing when $f'(x) > 0$	**M1**
	Attempts to find the points where the gradient is zero. $(3x + 7)(x - 5) = 0$ (or attempts to solve quadratic inequality)	**M1**
	$x = -\frac{7}{3}$ and $x = 5$, so f(x) is increasing when $\{x : x < -\frac{7}{3}\} \cup \{x : x > 5\}$ (or $x < -\frac{7}{3}$ or $x > 5$)	**A1**
	NOTE: Allow other method to find critical value (e.g. formula or calculator). This may be implied by correct answers. Correct notation ("or" or "$\cup$") must be seen for final A mark.	**Total 5 marks**

6	
Writes $\sqrt{t}$ as $t^{\frac{1}{2}}$ or $50\sqrt{t}$ as $50t^{\frac{1}{2}}$ (can be implied by correct integral).	**B1**
Makes an attempt to find $\frac{1}{20}\int(50t^{\frac{1}{2}} + 20t^2 - t^3)\mathrm{d}t$. Raising at least one t power by 1 would constitute an attempt.	**M1**
Makes a fully correct integration (ignore limits at this stage). $s = \frac{1}{20}\left[\frac{100}{3}t^{\frac{3}{2}} + \frac{20}{3}t^3 - \frac{t^4}{4}\right]_0^{20}$	**M1**
Makes an attempt to substitute the limits into their integrated function. For example, $\frac{1}{20}\left[\left(\frac{100}{3}\times 20^{\frac{3}{2}} + \frac{20\times 20^3}{3} - \frac{20^4}{4}\right) - \left(\frac{100}{3}\times 0^{\frac{3}{2}} + \frac{20\times 0^3}{3} - \frac{0^4}{4}\right)\right]$ is seen. Award mark even if the 0 limit is not shown.	**M1ft**
States fully correct answer. $s = 816$ cao.	**A1**
	Total 5 marks

7a	Statement that discriminant is $b^2 - 4ac$, and/or implied by writing $(k+8)^2 - 4\times1\times(8k+1)$	**M1**
	Attempt to simplify the expression by multiplying out the brackets. Condone sign errors and one algebraic error (but **not** missing k term from squaring brackets and must have k^2, k and constant terms). $k^2 + 8k + 8k + 64 - 32k - 4$ o.e.	**M1**
	$k^2 - 16k + 60$	**A1**
		(3 marks)
7b	Knowledge that two equal roots occur when the discriminant is zero. This can be shown by writing $b^2 - 4ac = 0$, or by writing $k^2 - 16k + 60 = 0$	**M1**
	$k = 10,\ k = 6$	**A1**
		(2 marks)
7c	Correct substitution for $k = 8$: $\mathrm{f}(x) = x^2 - 16x + 65$	**B1**
	Attempt to complete the square for their expression of f(x). $\mathrm{f}(x) = (x-8)^2 + 1$	**M1**
	<u>Statement</u> (which can be purely algebraic) that f(x) > 0, because, for example, a squared term is always greater than or equal to zero, so one more than a square term must be greater than zero or an appeal to a reasonable sketch of y = f(x).	**A1**
		(3 marks)
		Total 8 marks

NOTE:

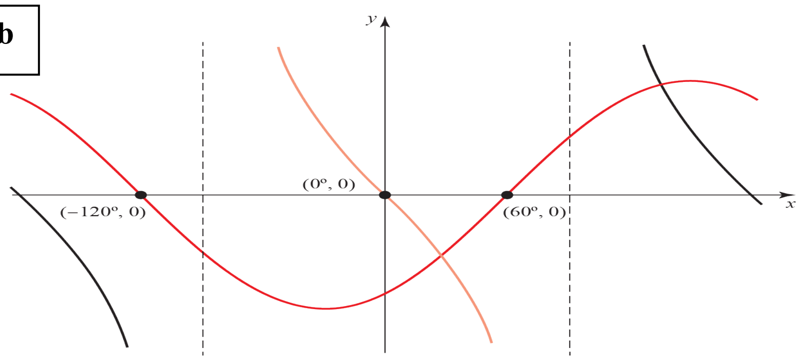
7a: Not all steps have to be present to award full marks. For example, the second method mark can still be awarded if the answer does not include that step.

7b: Award full marks for $k = 6$, $k = 10$ seen. Award full marks for valid and complete alternative method (e.g. expanding $(x - a)^2$ comparing coefficients and solving for k).

7c: An alternative method is acceptable. For example, students could differentiate to find that the turning point of the graph of y = f(x) is at (8, 1), and then show that it is a minimum. The minimum can be shown by using properties of quadratic curves or by finding the second differential. Students must explain (a sketch will suffice) that this means that the graph lies above the x-axis and reach the appropriate conclusion.

8a	Student attempts to complete the square twice for the first equation (condone sign errors). $(x+5)^2 - 25 + (y-6)^2 - 36 = 3$ $(x+5)^2 + (y-6)^2 = 64$	**M1**
	Centre $(-5, 6)$	**A1**
	Radius = 8	**A1**
	Student attempts to complete the square twice for the second equation (condone sign errors). $(x-3)^2 - 9 + (y-q)^2 - q^2 = 9$ $(x-3)^2 + (y-q)^2 = 18 + q^2$	**M1**
	Centre $(3, q)$	**A1**
	Radius = $\sqrt{18+q^2}$	**A1**
		(6 marks)
8b	Uses distance formula for their centres and $\sqrt{80}$. For example, $(-5-3)^2 + (6-q)^2 = \left(\sqrt{80}\right)^2$	**M1**
	Student simplifies to 3 term quadratic. For example, $q^2 - 12q + 20 = 0$	**M1**
	Concludes that the possible values of q are 2 and 10	**A1**
		(3 marks)
		Total 9 marks

9a	Substitutes (2, 400) into the equation. $400 = ab^2$	**M1**
	Substitutes (5, 50) into the equation. $50 = ab^5$	**M1**
	Makes an attempt to solve the expressions by division. For example, $b^3 = \frac{1}{8}$ (or equivalent) seen.	**M1**
	Solves for b. $b = 0.5$ or $b = \frac{1}{2}$	**A1**
	Solves for a. $a = 1600$	**A1**
9b		**(5 marks)**
	Divides by '1600' and takes logs of both sides. $\log\left(\frac{1}{2}\right)^x < \log\left(\frac{k}{1600}\right)$	**M1ft**
	Uses the third law of logarithms to write $\log\left(\frac{1}{2}\right)^x = x\log\left(\frac{1}{2}\right)$ or $\log 2^x = x\log 2$ anywhere in solution.	**B1**
	Uses the law(s) of logarithms to write $\log\left(\frac{1}{2}\right) = -\log 2$ anywhere in solution.	**B1**
	Uses above to obtain $x > \frac{\log\left(\frac{1600}{k}\right)}{\log 2}$ *	**A1***
		(4 marks)
		Total 9 marks

10a	$-2\sqrt{3}$ or awrt -3.46	**B1**
		(1 mark)
10b	 Sine curve with max 2 and min -2 Sine curve translated 60° to the right. Sin curve cuts x-axis at $(-120°, 0)$ and $(60°, 0)$ and the y-axis $(0, -\sqrt{3})$. Asymptotes for tan curve at $x = 90°$ and $x = -90°$ Tangent curve is 'flipped'. Uses the value of $-2\tan(-120°)$ to deduce no intersection in 3rd quadrant (can be implied). Tangent curve cuts x-axis at $(-180°, 0)$,$(0, 0)$ and$(180°, 0)$.	**B1** **B1** **B1** **B1** **B1** **B1** **B1**
		(7 marks)
10c	States that solutions to the equation $2\sin(x-60^{\circ})+\tan x=0$ will occur where the two curves intersect.	**B1 f.t.**
		(1 mark)
10d	States that there are two solutions in the given interval.	**B1**
		(1 mark)
		Total 10 marks

NOTES:

10b: Ignore any portion of curve(s) outside $-180° \leqslant x \leqslant 180°$

10c: Award both marks for correctly stating that there are two solutions even if explanation is missing.

11a	Attempts to differentiate.	**M1**
	$\frac{dy}{dx} = 3x^2 - 2x - 1$	**A1**
		(2 marks)
11b	Substitutes into equation for C to find y-coordinate. $x = 2, y = 2^3 - 2^2 - 2 + 2 = 4$	**M1**
	Substitutes $x = 2$ into $f'(x)$ to find gradient of tangent. $\frac{dy}{dx} = 3(4) - 2(2) - 1 = 7$	**M1**
	Finds equation of tangent using $y - y_1 = m(x - x_1)$ with (2, 4) $y - 4 = 7(x - 2)$	**M1**
	$y = 7x - 10$ o.e.	**A1**
		(4 marks)
11c	States or implies gradient of tangent is 7, so gradient of normal is $-\frac{1}{7}$	**M1**
	Finds equation of normal using $y - y_1 = m(x - x_1)$ with (2, 4) $y - 4 = -\frac{1}{7}(x - 2)$	**M1**
	Substitutes $y = 0$ and attempts to solve for x.	**M1**
	$x = 30$, $A(30, 0)$	**A1**
		(4 marks)
		Total 10 marks

NOTES:

11b: Using $y = mx + c$ is acceptable. For example $4 = 7 \times 2 + c$, so $c = -10$

11c: Using $y = mx + c$ is acceptable. For example $4 = \left(-\frac{1}{7}\right)(2) + c$, so $c = \frac{30}{7}$

12a	Makes an attempt to interpret the meaning of $f(5) = 0$. For example, writing $125 + 25 + 5p + q = 0$	**M1**
	$5p + q = -150$	**A1**
	Makes an attempt to interpret the meaning of $f(-3) = 8$. For example writing $-27 + 9 - 3p + q = 8$	**M1**
	$-3p + q = 26$	**A1**
	Makes an attempt to solve the simultaneous equations.	**M1ft**
	Solves the simultaneous equations to find that $p = -22$	**A1ft**
	Substitutes their value for p to find that $q = -40$	**A1ft**
		(7 marks)
12b	Draws the conclusion that $(x - 5)$ must be a factor.	**M1**
	Either makes an attempt at long division by setting up the long division, or makes an attempt to find the remaining factors by matching coefficients. For example, stating: $(x-5)(ax^2+bx+c)=x^3+x^2-22x-40$ (ft their −22 or −40)	**M1ft**
	For the long division, correctly finds the the first two coefficients. For the matching coefficients method, correctly deduces that $a = 1$ and $c = 8$	**A1**
	For the long division, correctly completes all steps in the division. For the matching coefficients method, correctly deduces that $b = 6$	**A1**
	States a fully correct, fully factorised final answer: $(x - 5)(x + 4)(x + 2)$	**A1**
		(5 marks)
		Total 12 marks

NOTES: 12a: Award ft through marks for correct attempt/answers to solving their simultaneous equations.

12b: Other algebraic methods can be used to factorise: $x - 5$ is a factor (M1)

$x^3 - x^2 - 22x - 40 = x^2(x-5) + 6x(x-5) + 8(x-5)$ by balancing (M1)

$= (x^2 + 6x + 8)(x-5)$ by factorising (M1)

$= (x+4)(x+2)(x-5)$ by factorising (A1 A1) (i.e. A1 for each factor other than $(x - 5)$)

13a Shows how to move from M to N using vectors. $\overrightarrow{MN} = \overrightarrow{MB} + \overrightarrow{BC} + \overrightarrow{CN} = \frac{4}{5}\mathbf{b} + \mathbf{a} - \frac{1}{5}\mathbf{b}$ or $\overrightarrow{MN} = \overrightarrow{MO} + \overrightarrow{OA} + \overrightarrow{AN} = -\frac{1}{5}\mathbf{b} + \mathbf{a} + \frac{4}{5}\mathbf{b}$	**M1**
$\overrightarrow{MN} = \mathbf{a} + \frac{3}{5}\mathbf{b}$	**A1**
	(2 marks)
13b Shows how to move from S to T using vectors. $\overrightarrow{ST} = \overrightarrow{SB} + \overrightarrow{BO} + \overrightarrow{OT} = -\frac{1}{5}\mathbf{a} - \mathbf{b} + \frac{4}{5}\mathbf{a}$ or $\overrightarrow{ST} = \overrightarrow{SC} + \overrightarrow{CA} + \overrightarrow{AT} = \frac{4}{5}\mathbf{a} - \mathbf{b} - \frac{1}{5}\mathbf{a}$	**M1**
$\overrightarrow{ST} = \frac{3}{5}\mathbf{a} - \mathbf{b}$	**A1**
	(2 marks)
13c Finds $\overrightarrow{OD}$ travelling via M. $\overrightarrow{OD} = \overrightarrow{OM} + \overrightarrow{MD} = \frac{1}{5}\mathbf{b} + \lambda\left(\mathbf{a} + \frac{3}{5}\mathbf{b}\right)$	**M1***
Finds $\overrightarrow{OD}$ travelling via T. $\overrightarrow{OD} = \overrightarrow{OT} + \overrightarrow{TD} = \frac{4}{5}\mathbf{a} + \mu\left(-\frac{3}{5}\mathbf{a} + \mathbf{b}\right)$	**M1***
Recognises that any two ways of travelling from O to D must be equal and equates $\overrightarrow{OD}$ via M with $\overrightarrow{OD}$ via T. $\frac{1}{5}\mathbf{b} + \lambda\left(\mathbf{a} + \frac{3}{5}\mathbf{b}\right) = \frac{4}{5}\mathbf{a} + \mu\left(-\frac{3}{5}\mathbf{a} + \mathbf{b}\right)$ or $\lambda\mathbf{a} + \left(\frac{1}{5} + \frac{3}{5}\lambda\right)\mathbf{b} = \left(\frac{4}{5} - \frac{3}{5}\mu\right)\mathbf{a} + \mu\mathbf{b}$	**M1***
Equates the **a** parts: $\lambda = \frac{4}{5} - \frac{3}{5}\mu$ or $5\lambda = 4 - 3\mu$ or $3\mu + 5\lambda = 4$	**M1***
Equates the **b** parts: $\frac{1}{5} + \frac{3}{5}\lambda = \mu$ or $1 + 3\lambda = 5\mu$ or $5\mu - 3\lambda = 1$	**M1***
Makes an attempt to solve the pair of simultaneous equations by multiplying. For example, $15\mu + 25\lambda = 20$ and $15\mu - 9\lambda = 3$ or $9\mu + 15\lambda = 12$ and $25\mu - 15\lambda = 5$	**M1**
Solves to find $\lambda = \frac{1}{2}$ and $\mu = \frac{1}{2}$	**A1**
Either: explains, making reference to an expression for $\overrightarrow{OD}$ or, for example, $\overrightarrow{MD}$ that $\lambda = \frac{1}{2}$ implies that D is the midpoint of MN or finds $\overrightarrow{MD} = \overrightarrow{DN}$ or $\overrightarrow{MD} = \frac{1}{2}\overrightarrow{MN}$ o.e. and therefore MN is bisected by ST.	**B1**
Uses argument (as above) for bisection of ST using $\mu = \frac{1}{2}$	**B1**
	9 marks)

	Total 13 marks

NOTES:

13c: Equating, for example, $\overrightarrow{OD}$ via M with $\overrightarrow{OD}$ via N, will lead to a pair of simultaneous equations that has infinitely many solutions. In this case, providing all work is correct, award one of the first two method marks, together with the 3rd, 4th, 5th and 6th method marks, for a maximum of 5 out of 9.

<u>Alternative Method</u>

(M1) Finds $\overrightarrow{OD}$ travelling via N.

$$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AN} + \overrightarrow{ND} = \mathbf{a} + \frac{4}{5}\mathbf{b} + \lambda\left(-\mathbf{a} - \frac{3}{5}\mathbf{b}\right)$$

(M1) Finds $\overrightarrow{OD}$ travelling via S.

$$\overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{BS} + \overrightarrow{SD} = \mathbf{b} + \frac{1}{5}\mathbf{a} + \mu\left(\frac{3}{5}\mathbf{a} - \mathbf{b}\right)$$

(M1) Equates $\overrightarrow{OD}$ via N with $\overrightarrow{OD}$ via S.

$$\mathbf{a} + \frac{4}{5}\mathbf{b} + \lambda\left(-\mathbf{a} - \frac{3}{5}\mathbf{b}\right) = \mathbf{b} + \frac{1}{5}\mathbf{a} + \mu\left(\frac{3}{5}\mathbf{a} - \mathbf{b}\right)$$

(M1) Equates the **a** parts:

$$1 - \lambda = \frac{1}{5} + \frac{3}{5}\mu \text{ or } 5 - 5\lambda = 1 + 3\mu \text{ or } 3\mu + 5\lambda = 4$$

(M1) Equates the **b** parts:

$$\frac{4}{5} - \frac{3}{5}\lambda = 1 - \mu \text{ or } 4 - 3\lambda = 5 - 5\mu \text{ or } 5\mu - 3\lambda = 1$$

Proceeds as above.

(TOTAL: 100 MARKS)